Random-energy model in random fields

Luiz O. de Oliveira Filho,[1,*] Francisco Alexandre da Costa,[2] and Carlos S. O. Yokoi[1]
[1]*Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, SP, Brazil*
[2]*Departamento de Física Teórica e Experimental, Universidade Federal do Rio Grande do Norte, Caixa Postal 1641, 59072-970 Natal, RN, Brazil*

(Received 24 January 2006; published 15 September 2006)

The random-energy model is studied in the presence of random fields. The problem is solved exactly both in the microcanonical ensemble, without recourse to the replica method, and in the canonical ensemble using the replica formalism. The phase diagrams for bimodal and Gaussian random fields are investigated in detail. In contrast to the Gaussian case, the bimodal random field may lead to a tricritical point and a first-order transition. An interesting feature of the phase diagram is the possibility of a first-order transition from paramagnetic to mixed phase.

DOI: 10.1103/PhysRevE.74.031117 PACS number(s): 05.50.+q, 75.10.Nr, 64.60.−i

I. INTRODUCTION

Spin-glass [1,2] and random-field models [3] have played prominent roles in the study of disordered systems in the last few decades. Although the random-exchange and random-field effects are are usually considered separately, it has been argued that in proton glasses such as $Rb_{1-x}(NH_4)_xH_2PO_4$ [4] it is necessary to take into account the effect of random fields generated by the presence of impurities. Another example where the spin-glass and random-field effects are present simultaneously is the diluted antiferromagnets $Fe_xZn_{1-x}F_2$ [5,6].

The effect of random fields on the well known Sherrington-Kirkpatrick (SK) model for spin glass [7] has been investigated for Gaussian random fields [8], bimodal random fields [9], and trimodal random fields [10]. These studies, limited to the replica-symmetric solution, indicate that the bimodal and trimodal random fields, but not the Gaussian random fields, may induce first-order phase transitions and tricritical points. Since the full replica-symmetry-breaking solution of the SK model is rather difficult to work out explicitly [1,2], it seems worthwhile to consider a simpler spin-glass model where the effect of random fields on the phase diagram can be investigated thoroughly.

The random-energy model (REM) [11,12] is probably the simplest spin-glass model [13] retaining some important properties of the SK model. The REM is related to the generalization of the SK model to include interaction between every set of p spins [13]. In the $p\to\infty$ limit the energies of the spin configurations become independent random variables and the model reduces to the REM.

In this paper we investigate the effect of random fields on the REM. The model is given as the $p\to\infty$ limit of the Hamiltonian

$$\mathcal{H} = -\sum_{i_1<\cdots<i_p} J_{i_1\ldots i_p} S_{i_1}\cdots S_{i_p} - J_0\sum_{i<j} S_iS_j - \sum_i H_iS_i, \quad (1)$$

where $S_i=\pm 1$ are Ising spins, $J_{i_1\ldots i_p}$ are independent quenched Gaussian random couplings with zero mean and variance $p!J^2/2N^{p-1}$, $J_0\geqslant 0$ are ferromagnetic couplings, and H_i are independent identically distributed quenched random fields.

The Hamiltonian (1) for $p=2$ is the SK model in a random field, whereas for $p\to\infty$ it reduces to the REM model in a random field. We have solved the problem exactly by two complementary approaches. In Sec. II we employ the microcanonical formalism [12] to obtain the thermodynamic quantities directly. In Sec. III we employ the replica formalism [13] to determine the spin-glass order parameters. In Sec. IV we study the phase diagram for bimodal and Gaussian distribution of random fields. Finally, in Sec. V we compare our results with the previous studies on related models and make some concluding remarks.

II. MICROCANONICAL APPROACH

In this section we solve the model in the microcanonical ensemble [12]. Let $S=(S_1,\ldots,S_N)$ denote one of 2^N spin configurations or the microstates of the system. The energy of a given microstate is given by

$$E_S = \mathcal{H}(S) = -\sum_{i_1<\cdots<i_p} J_{i_1\cdots i_p} S_{i_1}\cdots S_{i_p} + \mathcal{H}_0(S), \quad (2)$$

where $\mathcal{H}_0$ denotes the part of the Hamiltonian without random couplings. Since E_S are linear combinations of Gaussian random variables $J_{i_1\cdots i_p}$, they are themselves Gaussian random variables with mean

$$\langle E_S\rangle = \mathcal{H}_0(S) = E_S^0, \quad (3)$$

and covariance

$$\sigma_{SS'} = \langle (E_S - E_S^0)(E_{S'} - E_{S'}^0)\rangle = \frac{J^2N}{2}\left[q_{SS'}^p + O\left(\frac{1}{N}\right)\right], \quad (4)$$

where

*Permanent address: Coordenação de Física, Universidade Estadual Vale do Acaraú, Av. Doutor Guarani 317, 62040-730 Sobral, CE, Brazil.

$$q_{SS'} = \frac{1}{N}\sum_i S_i S_i', \tag{5}$$

is the overlap between the microstates S and S'. In the thermodynamic limit, $N\to\infty$, the energies E_S and $E_{S'}$ of two macroscopically distinguishable microstates S and S' become uncorrelated in the $p\to\infty$ limit,

$$\sigma_{SS'} = \left(\frac{J^2N}{2}\right)q_{SS'}^p \to 0, \quad \text{for } p\to\infty \text{ and } |q_{SS'}|<1. \tag{6}$$

Thus, in the $p\to\infty$ limit the energies E_S become independent Gaussian random variables. The multivariate probability density is then the product of univariate probability densities given by

$$f_{E_S}(E) = \frac{1}{\sqrt{\pi NJ^2}}\exp\left[-\frac{(E-E_S^0)^2}{NJ^2}\right]. \tag{7}$$

Let us consider a given sample, that is, a particular realization of the random couplings $J_{i_1\cdots i_p}$. The entropy of the sample is given by

$$S(E) = k_B \ln \Omega(E), \tag{8}$$

where

$$\Omega(E) = \sum_S \delta(E-E_S) \tag{9}$$

is the density of states. The average density of states is

$$\langle\Omega(E)\rangle = \sum_S \langle\delta(E-E_S)\rangle = \sum_S f_{E_S}(E). \tag{10}$$

Due to the statistical independence of E_S, the fluctuations around this average are of order $\langle\Omega(E)\rangle^{-1/2}$, and are thus completely negligible [12].

We can rewrite the average density of states in the form

$$\langle\Omega(E)\rangle = \frac{1}{\sqrt{\pi NJ^2}}\int_{-\infty}^{\infty} dE_0 \exp\left[-\frac{(E-E_0)^2}{NJ^2}\right]\sum_S \delta(E_0-E_S^0). \tag{11}$$

We recognize

$$\Omega_0(E) = \sum_S \delta(E_0-E_S^0) \tag{12}$$

as the density of states of the system described by the Hamiltonian $\mathcal{H}_0$. Therefore,

$$\langle\Omega(E)\rangle = \frac{1}{\sqrt{\pi NJ^2}}\int_{-\infty}^{\infty} dE_0 \exp\left[-\frac{(E-E_0)^2}{NJ^2}+\frac{S_0(E_0)}{k_B}\right], \tag{13}$$

where

$$S_0(E_0) = k_B \ln \Omega_0(E_0) \tag{14}$$

is the entropy of the system characterized by the Hamiltonian $\mathcal{H}_0$. In the thermodynamic limit, $N\to\infty$, we have

$$\ln\langle\Omega(E)\rangle = \max_{E_0}\left[-\frac{(E-E_0)^2}{NJ^2}+\frac{S_0(E_0)}{k_B}\right]. \tag{15}$$

E_0 is determined by

$$\frac{1}{T_0(E_0)} = \frac{\partial S_0(E_0)}{\partial E_0} = -\frac{2k_B(E-E_0)}{NJ^2}, \tag{16}$$

where $T_0(E_0)$ is by definition the temperature of the system described by the Hamiltonian $\mathcal{H}_0$.

For energies E such that $\ln\langle\Omega(E)\rangle>0$ the average density of states is very large and the fluctuation is negligible. Thus, we have with probability 1,

$$S(E) = k_B\ln\langle\Omega(E)\rangle = -\frac{k_B(E-E_0)^2}{NJ^2}+S_0(E_0). \tag{17}$$

For energies E such that $\ln\langle\Omega(E)\rangle<0$, the average density of states is very small. Thus with probability 1 there are no samples with this energy.

The temperature of the system is given by

$$\frac{1}{T(E)} = \frac{\partial S(E)}{\partial E} = -\frac{2k_B(E-E_0)}{NJ^2}, \tag{18}$$

which coincides with the temperature of the system described by the Hamiltonian $\mathcal{H}_0$,

$$T(E) = T_0(E_0). \tag{19}$$

Therefore the energy of the system as a function of temperature is given by

$$E(T) = E_0(T) - \frac{NJ^2}{2k_BT}, \tag{20}$$

where $E_0(T)$ is the energy of the system characterized by the Hamiltonian $\mathcal{H}_0$. The entropy as a function of the temperature is

$$S(T) = S_0(T) - \frac{NJ^2}{4k_BT^2}. \tag{21}$$

These results are valid above a critical temperature T_c determined by

$$S(T_c) = S_0(T_c) - \frac{NJ^2}{4k_BT_c^2} = 0. \tag{22}$$

Below this temperature the system is frozen in its ground state.

These results are valid for any Hamiltonian $\mathcal{H}_0$. We now particularize for the case where the Hamiltonian $\mathcal{H}_0$ describes the Ising model with infinite range ferromagnetic interactions in a random field [14–16],

$$\mathcal{H}_0 = -\frac{J_0}{N}\sum_{i<j} S_iS_j - \sum_i H_iS_i = -\frac{J_0}{2N}\left(\sum_i S_i\right)^2 - \sum_i H_iS_i, \tag{23}$$

where in the last passage we have dropped the term $J_0/2$ that is negligible in the thermodynamic limit. The quadratic term can be linearized using the identity

$$e^{\lambda a^2/2} = \sqrt{\frac{\lambda}{2\pi}}\int_{-\infty}^{\infty} dx e^{-\lambda x^2/2+\lambda a x}, \tag{24}$$

and the partition function will be given by

$$Z_0 = \sum_S e^{-\beta\mathcal{H}_0} = \sqrt{\frac{\beta J_0 N}{2\pi}}\int_{-\infty}^{\infty} dm \exp\left\{N\left[-\frac{1}{2}\beta J_0 m^2 + \frac{1}{N}\sum_i \ln 2\cosh\beta(J_0 m + H_i)\right]\right\}. \tag{25}$$

In the thermodynamic limit, $N\to\infty$, the Laplace method gives

$$\ln Z_0 = N\max_m\left[-\frac{1}{2}\beta J_0 m^2 + \langle \ln 2\cosh\beta(J_0 m + H)\rangle\right], \tag{26}$$

where we have used the law of large numbers to write

$$\frac{1}{N}\sum_i \ln 2\cosh\beta(J_0 m + H_i) = \langle \ln 2\cosh\beta(J_0 m + H)\rangle, \tag{27}$$

where $\langle\cdots\rangle$ denotes the expectation value with respect to the random fields H. Thus the free energy is given by

$$F_0 = -\beta^{-1}\ln Z_0 = N\left[\frac{1}{2}J_0 m^2 - \frac{1}{\beta}\langle \ln 2\cosh\beta(J_0 m + H)\rangle\right], \tag{28}$$

where the magnetization m is determined by the equation

$$m = \langle \tanh\beta(J_0 m + H)\rangle. \tag{29}$$

The internal energy $E_0(T)$ and the entropy $S_0(T)$ follow from usual thermodynamic relations.

Applying the general results obtained previously for the system described by the full Hamiltonian (1) in the $p\to\infty$ limit, we obtain for the internal energy

$$\frac{E}{N} = -\frac{\beta J^2}{2} - \frac{1}{2}J_0 m^2 - \langle H\tanh\beta(J_0 m + H)\rangle, \tag{30}$$

and for the entropy

$$\frac{S}{Nk_B} = -\frac{(\beta J)^2}{4} - \beta J_0 m^2 - \beta\langle H\tanh\beta(J_0 m + H)\rangle + \langle \ln 2\cosh\beta(J_0 m + H)\rangle. \tag{31}$$

These results are valid for $\beta < \beta_c$ where β_c is determined by

$$S(\beta_c) = -\frac{(\beta_c J)^2}{4} - \beta_c\langle (H + J_0 m)\tanh\beta_c(H + J_0 m)\rangle + \langle \ln 2\cosh\beta_c(H + J_0 m)\rangle = 0 \tag{32}$$

and

$$m = \langle \tanh\beta_c(J_0 m + H)\rangle. \tag{33}$$

For $\beta > \beta_c$ the system is frozen in its ground state. Therefore

$$E(\beta) = E(\beta_c), \quad S(\beta) = 0. \tag{34}$$

III. REPLICA APPROACH

In this section we solve the model in the canonical ensemble [13]. We use the replica identity for the free energy

$$-\beta F = \langle \ln Z\rangle = \lim_{n\to 0}\frac{\langle Z^n\rangle - 1}{n} \tag{35}$$

to perform the average over the random couplings $J_{i_1 i_2\cdots i_p}$. To evaluate $\langle Z^n\rangle$ we introduce n replicas of the system $\alpha = 1,2,\ldots,n$,

$$\langle Z^n\rangle = \mathrm{Tr}\left\langle e^{-\beta\sum_{\alpha=1}^{n}\mathcal{H}(S^\alpha)}\right\rangle = \mathrm{Tr}\, e^{-\beta\mathcal{H}_{\mathrm{eff}}}, \tag{36}$$

where $\mathcal{H}_{\mathrm{eff}}$ denotes the effective Hamiltonian that results after taking the average over random couplings,

$$-\beta\mathcal{H}_{\mathrm{eff}} = \frac{N(\beta J)^2}{2}\left[\sum_{\alpha<\beta}\left(\frac{1}{N}\sum_i S_i^\alpha S_i^\beta\right)^p + \frac{n}{2}\right] + \frac{N\beta J_0}{2}\sum_\alpha\left(\frac{1}{N}\sum_i S_i^\alpha\right)^2 + \beta\sum_i H_i\sum_\alpha S_i^\alpha. \tag{37}$$

We have dropped terms that vanish in the thermodynamic limit, $N\to\infty$. The nonlinear terms can be linearized with the help of the asymptotic relation

$$e^{N\lambda f(a)} \sim \sqrt{\frac{N\lambda f''(a)}{2\pi}}\int_{-\infty}^{\infty} dx e^{N\lambda[f(x)-f'(x)(x-a)]}, \tag{38}$$

which can be proved for $\lambda f''(a) > 0$ and $N\to\infty$ applying the Laplace method. In particular for $f(x)=x^2/2$ the asymptotic relation reduces to the identity (24). Omitting the factors that do not contribute to the free energy in the thermodynamic limit, $N\to\infty$, we arrive at

$$\langle Z^n\rangle \sim \int \prod_{\alpha<\beta} dq_{\alpha\beta}\int\prod_\alpha dm_\alpha e^{-\beta F_n(q_{\alpha\beta},m_\alpha)}, \tag{39}$$

where

$$\frac{F_n}{N} = -\frac{1}{4}\beta J^2 n + \frac{1}{2}\beta J^2(p-1)\sum_{\alpha<\beta} q_{\alpha\beta}^p + \frac{1}{2}J_0\sum_\alpha m_\alpha^2 - \beta^{-1}\frac{1}{N}\sum_i \ln\mathrm{Tr}\exp\left[\frac{1}{2}(\beta J)^2 p\sum_{\alpha<\beta} q_{\alpha\beta}^{p-1}S^\alpha S^\beta + \beta\sum_\alpha (H_i + J_0 m_\alpha)S^\alpha\right]. \tag{40}$$

In the $N\to\infty$ limit we use the law of large numbers to write the last term as an expectation value over the random-field distribution and use the Laplace method to evaluate the integral. The free energy is then given by the stationary value of the functional

$$\frac{F}{N}=-\frac{1}{4}\beta J^2+\lim_{n\to 0}\frac{1}{n}\Bigg\{\frac{1}{2}\beta J^2(p-1)\sum_{\alpha<\beta}q_{\alpha\beta}^p+\frac{J_0}{2}\sum_\alpha m_\alpha^2 -\beta^{-1}\Bigg\langle \ln \mathrm{Tr}\exp\Bigg[\frac{1}{2}(\beta J)^2 p\sum_{\alpha<\beta}q_{\alpha\beta}^{p-1}S^\alpha S^\beta +\beta\sum_\alpha (H+J_0 m_\alpha)S^\alpha\Bigg]\Bigg\rangle\Bigg\}, \tag{41}$$

where $\langle\cdots\rangle$ denotes the expectation value with respect to the random field H.

To compute the free energy we assume

$$m_\alpha = m \tag{42}$$

to be independent of replica indices, and parameterize $q_{\alpha\beta}$ following the Parisi's K-step replica-symmetry-breaking ansatz [17]. In the $n\to 0$ limit the free energy functional becomes a function of the magnetization m and the parameters

$$0\le q_0\le q_1\le \cdots \le q_{K-1}\le q_K\le 1 \tag{43}$$

and

$$0=m_0\le m_1\le\cdots\le m_K\le m_{K+1}=1, \tag{44}$$

and is given by

$$\frac{F}{N}=-\frac{\beta J^2}{4}\left[1+(p-1)\sum_{i=0}^{K}(m_{i+1}-m_i)q_i^p-pq_K^{p-1}\right]+\frac{J_0}{2}m^2 -\int_{-\infty}^{\infty}dy\langle G_{\sigma_0^2}(y-H-J_0 m)\rangle g_0(y), \tag{45}$$

where $g_0(y)$ is given recursively by

$$g_{i-1}(y)=\frac{1}{\beta m_i}\ln\left\{\int_{-\infty}^{\infty}dy' G_{\sigma_i^2}(y'-y)\exp[\beta m_i g_i(y')]\right\}, \tag{46}$$

for $i=1,\ldots,K$ with the initial condition

$$g_K(y)=\frac{1}{\beta}\ln(2\cosh\beta y). \tag{47}$$

$G_{\sigma^2}(y)$ denotes the Gaussian distribution function

$$G_{\sigma^2}(y)=\frac{1}{J\sigma\sqrt{2\pi}}\exp\left(-\frac{y^2}{2J^2\sigma^2}\right), \tag{48}$$

where the variances σ_i^2 are given by

$$\sigma_0^2=\frac{p}{2}q_0^{p-1},\quad \sigma_i^2=\frac{p}{2}(q_i^{p-1}-q_{i-1}^{p-1})\quad \text{for } i=1,\ldots,K. \tag{49}$$

We first assume that all the q's are less than one, $0\le q_0\le\cdots\le q_{K-1}\le q_K<1$. Then $\sigma_i^2\to 0$ when $p\to\infty$ for $i=0,\ldots,K$. Using the expansion

$$\int_{-\infty}^{\infty}dy' G_{\sigma^2}(y'-y)f(y') =\exp\left(\frac{J^2\sigma^2}{2}\frac{d^2}{dy^2}\right)f(y)=1+\frac{J^2\sigma^2}{2}f''(y)+O(\sigma^4), \tag{50}$$

we obtain

$$\frac{F}{N}=-\frac{\beta J^2}{4}\left[1+(p-1)\sum_{i=0}^{K}(m_{i+1}-m_i)q_i^p-pq_K^{p-1}\right] +\frac{J_0}{2}m^2-\frac{1}{\beta}\langle\ln 2\cosh\beta(H+J_0m)\rangle -\sum_{i=0}^{K}\frac{\beta(J\sigma_i)^2}{2}[1-(1-m_i)\langle\tanh^2\beta(H+J_0m)\rangle] +O(\sigma_0^4,\ldots,\sigma_K^4,\sigma_0^2\sigma_1^2,\ldots,\sigma_0^2\sigma_K^2). \tag{51}$$

Stationarity of the free energy with respect to the variational parameters gives, in the limit $p\to\infty$,

$$m=\langle\tanh\beta(H+J_0m)\rangle \tag{52}$$

and

$$q_0=q_1=\cdots=q_K=\langle\tanh^2\beta(H+J_0m)\rangle. \tag{53}$$

Thus we arrived at the replica-symmetric solution where all the q's are identical. The free energy in the $p\to\infty$ limit is given by

$$\frac{F}{N}=-\frac{\beta J^2}{4}+\frac{J_0}{2}m^2-\frac{1}{\beta}\langle\ln 2\cosh\beta(H+J_0m)\rangle. \tag{54}$$

The entropy is

$$\frac{S}{Nk_B}=-\frac{(\beta J)^2}{4}-\beta\langle(J_0m+H)\tanh\beta(J_0m+H)\rangle +\langle\ln 2\cosh\beta(J_0m+H)\rangle. \tag{55}$$

This solution corresponds precisely to the high-temperature solution found in the microcanonical approach. Since the entropy becomes negative at low temperatures, it is necessary to consider a different solution for low temperatures.

We therefore assume that $0\le q_0\le\cdots\le q_{K-1}<q_K=1$. Then $\sigma_i^2\to 0$ for $i=0,\ldots,K-1$ and $\sigma_K\to\infty$ in the limit $p\to\infty$. A simple calculation yields

$$g_{K-1}(y)=\frac{1}{\beta m_K}\ln(2\cosh\beta m_K y)+\frac{1}{2}\beta m_K(J\sigma_K)^2 +O(e^{-(\beta J\sigma_K m_K)^2/2}\sigma_K^{-1}). \tag{56}$$

The error is exponentially small and may be safely ignored. The rest of calculation proceeds as before using the expansion (50) and we arrive at

$$\frac{F}{N} = -\frac{\beta J^2}{4}\left[1 + (p-1)\sum_{i=0}^{K}(m_{i+1}-m_i)q_i^p - pq_K^{p-1}\right] + \frac{J_0}{2}m^2$$
$$-\frac{1}{2}\beta m_K(J\sigma_K)^2 - \sum_{i=0}^{K-1}\frac{\beta(J\sigma_i)^2}{2}[m_K\langle\text{sech}^2\,\beta m_K(H+J_0m)\rangle$$
$$+ m_i\langle\tanh^2\beta m_K(H+J_0m)\rangle]$$
$$-\frac{1}{\beta m_K}\langle\ln 2\cosh\beta m_K(H+J_0m)\rangle$$
$$+ O(\sigma_0^4,\ldots,\sigma_{K-1}^4,\sigma_0^2\sigma_1^2,\ldots,\sigma_0^2\sigma_{K-1}^2). \quad (57)$$

Stationarity with respect to the variational parameters gives, in the limit $p\to\infty$,

$$m = \langle\tanh\beta m_K(H+J_0m)\rangle, \quad (58)$$

$$q_0 = q_1 = \cdots = q_{K-1} = \langle\tanh^2\beta m_K(H+J_0m)\rangle, \quad q_K = 1, \quad (59)$$

consistent with initial assumption $q_K=1$, and

$$\frac{(\beta J)^2}{4}m_K^2 = \langle\ln 2\cosh\beta m_K(H+J_0m)\rangle$$
$$-\beta m_K\langle(H+J_0m)\tanh\beta m_K(H+J_0m)\rangle. \quad (60)$$

These results are the same for all $K\geqslant 1$, showing that no other solutions are possible beyond one-step replica-symmetry breaking. The free energy in the limit $p\to\infty$ is given by

$$\frac{F}{N} = -\frac{\beta J^2}{4}m_K + \frac{J_0}{2}m^2 - \frac{1}{\beta m_K}\langle\ln 2\cosh\beta m_K(H+J_0m)\rangle. \quad (61)$$

The entropy is

$$\frac{S}{Nk_B} = -\frac{(\beta J)^2}{4}m_K - \beta\langle(J_0m+H)\tanh\beta m_K(J_0m+H)\rangle$$
$$+\frac{1}{m_K}\langle\ln 2\cosh\beta m_K(J_0m+H)\rangle. \quad (62)$$

Taking into account the self-consistency equation (60), we find that the entropy vanishes identically. Thus this solution corresponds to the frozen phase found in the microcanonical approach.

The self-consistency equations (58) and (60) imply that βm_K is independent of temperature. Since this solution is acceptable only for $m_K\leqslant 1$, we have

$$\beta m_K = \beta_c, \quad (63)$$

where β_c is found from the equations

$$\frac{(\beta_c J)^2}{4} = \langle\ln 2\cosh\beta_c(H+J_0m)\rangle$$
$$-\beta_c\langle(H+J_0m)\tanh\beta_c(H+J_0m)\rangle \quad (64)$$

and

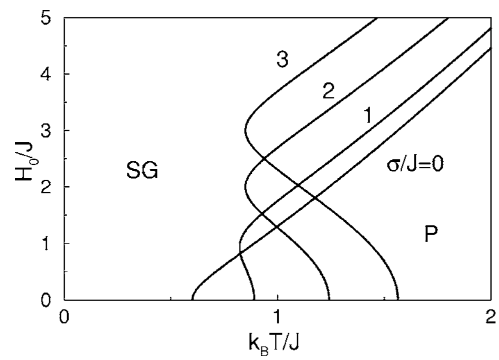

FIG. 1. The $J_0=0$ phase diagram for a bimodal random field for various values of σ.

$$m = \langle\tanh\beta_c(H+J_0m)\rangle. \quad (65)$$

Thus we see that β_c corresponds precisely to the critical temperature for the transition to the frozen phase found in the microcanonical approach. The Parisi order parameter function $q(x)$ [17] has two flat portions $q_0=m^2$ and $q_K=1$, with a discontinuous jump at $x=m_K=T/T_c$,

$$q(x) = m^2\theta\left(\frac{T}{T_c}-x\right) + \theta\left(x-\frac{T}{T_c}\right). \quad (66)$$

The overlap distribution function $P(q)$ [18] is given by

$$P(q) = \frac{T}{T_c}\delta(q-m^2) + \left(1-\frac{T}{T_c}\right)\delta(q-1). \quad (67)$$

Thus the frozen phase is indeed a spin-glass phase with many pure states having minimal overlap between them and maximal self-overlap [13].

IV. PHASE DIAGRAMS

The phase diagrams for the model defined by the Hamiltonian (1) in the $p\to\infty$ limit were determined for two distributions of random fields which are often considered in the literature: The discrete bimodal distribution [15]

$$P(H) = \frac{1}{2}\delta(H-H_0+\sigma) + \frac{1}{2}\delta(H-H_0-\sigma) \quad (68)$$

and the continuous Gaussian distribution function [14]

$$P(H) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(H-H_0)^2/2\sigma^2}. \quad (69)$$

In both cases the means and variances are H_0 and σ^2, respectively.

The $H_0\times T$ phase diagrams in the absence of ferromagnetic interactions ($J_0=0$) and various values of the standard deviation σ are shown in Fig. 1 and Fig. 2 for bimodal and Gaussian distributions, respectively. Notice that the case $\sigma=0$ reduces to the REM in a uniform field [12]. There are two phases: replica-symmetric paramagnetic phase (P) and a frozen spin-glass (SG) phase with one-step replica-symmetry breaking. The transitions are determined by Eq. (32) for $J_0=0$,

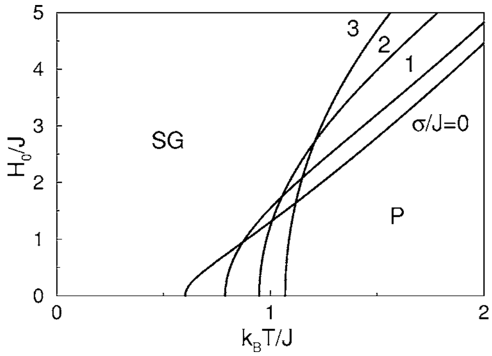

FIG. 2. The $J_0=0$ phase diagram for a Gaussian random field for various values of σ.

$$-\frac{(\beta_c J)^2}{4} - \beta_c \langle H \tanh \beta_c H \rangle + \langle \ln 2 \cosh \beta_c H \rangle = 0. \quad (70)$$

These transitions are second order in the thermodynamic sense, but the Parisi order parameter (66) changes discontinuously at the transition. For both distributions the transition temperature is depressed for high values of the bias field H_0 and increased for small H_0. The effect is most pronounced for the bimodal case where the changeover occurs for H_0 very close to σ.

The $T \times J_0$ phase diagrams for symmetric random-field distributions ($H_0=0$) are shown in Figs. 3–5. For comparison the phase diagrams for the case $\sigma=0$, corresponding to the REM with ferromagnetic interactions, are shown by thin dashed lines. There are four phases: replica-symmetric paramagnetic (P) and ferromagnetic (F) phases, and frozen spin-glass (SG) and mixed (M) phases with one-step replica symmetry breaking. Unlike the SG phase, in the M phase there is a nonzero magnetization ($m \neq 0$).

The transition from the P to SG phase is determined by Eq. (32) for $m=0$, which is identical to Eq. (70). Since there is no dependence on J_0, it represents a horizontal line in the $T \times J_0$ phase diagram. This transition is second order in the thermodynamic sense, but the Parisi order parameter (66) changes discontinuously at the transition.

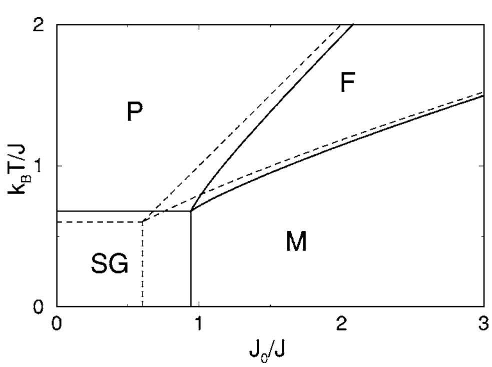

FIG. 3. The phase diagram for a symmetric bimodal random field. The results for $\sigma/J=0.4$ are shown by the solid curves. For comparison the results in the absence of random fields are shown by the dashed curves.

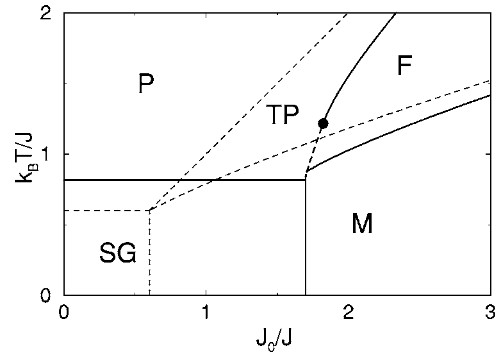

FIG. 4. The phase diagram for a symmetric bimodal random field. The results for $\sigma/J=0.8$ are shown by the solid curves (second-order transition) and thick dashed curves (first-order transition). For comparison the results in the absence of random fields are shown by the thin dashed curves.

A second-order transition from the P to F phase can be determined by expanding the equation of state,

$$m = \langle \tanh \beta(H + J_0 m) \rangle, \quad (71)$$

in powers of the magnetization m [15]. For symmetric distribution of the random fields ($H_0=0$) we find

$$m = am - bm^3 - cm^5 - \cdots, \quad (72)$$

where

$$a = \beta J_0 (1 - \langle \tanh^2 \beta H \rangle), \quad (73)$$

$$b = \frac{1}{3}(\beta J_0)^3 (1 - 4\langle \tanh^2 \beta H \rangle + 3\langle \tanh^4 \beta H \rangle), \quad (74)$$

$$c = -\frac{1}{15}(\beta J_0)^5 (2 - 17\langle \tanh^2 \beta H \rangle + 30\langle \tanh^4 \beta H \rangle - 15\langle \tanh^6 \beta H \rangle). \quad (75)$$

There is a second-order transition from the P to F phase for $a=1$ and $b>0$. For $b=0$ there is a tricritical point, and for $b<0$ the transition is first order and can only be determined numerically by equating the free energies of both phases.

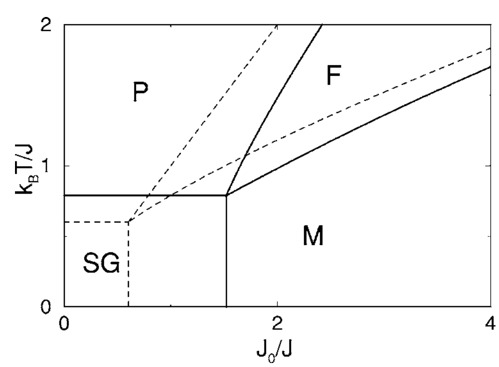

FIG. 5. The phase diagram for a symmetric Gaussian random field. The results for $\sigma/J=1$ are shown by the solid curves. For comparison the results in the absence of random fields are shown by the dashed curves.

For the bimodal distribution of random fields, the conditions $a=b=0$ give

$$\beta\sigma = \tanh^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{2}\ln(2+\sqrt{3}), \tag{76}$$

which determines the location of the tricritical point. This tricritical point occurs above the freezing transition (70) only if the standard deviation σ is greater than the threshold value

$$\frac{\sigma_c}{J} = \frac{1}{4}\ln(2+\sqrt{3})\left[\ln(3+\sqrt{3}) - \frac{1}{6}(3+\sqrt{3})\ln(2+\sqrt{3})\right]^{-1/2} = 0.4584695507\ldots. \tag{77}$$

Thus the phase diagrams for the bimodal distribution are qualitatively different depending on the value of standard deviation σ.

For $\sigma<\sigma_c$ the phase diagrams do not differ qualitatively from the case without random fields, as shown in Fig. 3. All the transition lines are second order in the thermodynamic sense. However, across the F-M and SG-M transitions the Parisi order parameter (66) changes discontinuously.

For $\sigma>\sigma_c$ the phase diagrams change qualitatively compared to the case without random fields. The second-order P-F transition line ends at a tricritical point below which the transition is of first order, shown by thick dashed line in Fig. 4. This line was determined by equating the free energies of the two neighboring phases. The transition is of first order in the thermodynamic sense as well as in the discontinuity of the Parisi order parameter (66) across the transition.

For the Gaussian distribution one always has $b>0$ when $a=1$. Thus the P-F transition is always of second order and the phase diagrams and the nature of the transitions do not differ qualitatively from the case of bimodal distribution for $\sigma<\sigma_c$, as shown in Fig. 5.

V. DISCUSSION

We solved exactly the REM in a random field in both microcanonical and replica approaches. We investigated in detail the phase diagrams for bimodal and Gaussian random-field distributions with mean H_0 and variance σ^2. The Gaussian random fields do not change the phase diagrams qualitatively. The bimodal random fields, on the contrary, change the $H_0=0$ phase diagram qualitatively for sufficiently large σ by leading to a tricritical point and a first-order transition at low temperatures. The same conclusions were reached in the replica-symmetric study of the SK model in a Gaussian random field [8] and bimodal random fields [9].

In the mean-field approximation, a ferromagnetic Ising model in a random field with minimum at zero field leads to a tricritical point and a first-order transition [15,16]. Our results indicate that that this property remains true even in the presence of random interactions, although at low temperatures the emergence of the tricritical point may be forestalled by the spin-glass phase. However, at present there is no conclusive evidence of this property in the three-dimensional short-ranged Ising model in random fields. In fact, Monte Carlo simulations detect a jump in the magnetization but no latent heat for both bimodal [19] and Gaussian [20] distributions, and high-temperature series expansions [21] find a continuous transition for both distributions.

Our results can shed useful light on the nature of phase diagram of more sophisticated spin-glass models with random fields [8–10]. One interesting feature of the phase diagram with a tricritical point is the possibility of a first-order transition from a paramagnetic to mixed a phase.

ACKNOWLEDGMENTS

This work was supported in part by the Brazilian Government Agencies CAPES and CNPq.

[1] K. Binder and A. P. Young, Rev. Mod. Phys. **58**, 801 (1986).
[2] M. Mézard, G. Parisi, and M. A. Virasoro, *Spin Glass Theory and Beyond* (World Scientific, Singapore, 1987).
[3] *Spin Glasses and Random Fields*, edited by A. P. Young (World Scientific, Singapore, 1998).
[4] R. Pirc, B. Tadic, and R. Blinc, Phys. Rev. B **36**, 8607 (1987).
[5] F. C. Montenegro, A. R. King, V. Jaccarino, S. J. Han, and D. P. Belanger, Phys. Rev. B **44**, 2155 (1991).
[6] S. R. Vieira, F. D. Nobre, and C. S. O. Yokoi, Phys. Rev. E **61**, 4760 (2000).
[7] D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. **35**, 1792 (1975).
[8] R. F. Soares, F. D. Nobre, and J. R. L. de Almeida, Phys. Rev. B **50**, 6151 (1994).
[9] E. Nogueira, F. D. Nobre, F. A. da Costa, and S. Coutinho, Phys. Rev. E **57**, 5079 (1998).
[10] J. M. de Araújo, F. D. Nobre, and F. A. da Costa, Phys. Rev. E **61**, 2232 (2000).
[11] B. Derrida, Phys. Rev. Lett. **45**, 79 (1980).
[12] B. Derrida, Phys. Rev. B **24**, 2613 (1981).
[13] D. J. Gross and M. Mezard, Nucl. Phys. B **240**, 431 (1984).
[14] T. Schneider and E. Pytte, Phys. Rev. B **15**, 1519 (1977).
[15] A. Aharony, Phys. Rev. B **18**, 3318 (1978).
[16] S. R. Salinas and W. F. Wreszinski, J. Stat. Phys. **41**, 299 (1985).
[17] G. Parisi, J. Phys. A **13**, L115 (1980).
[18] G. Parisi, Phys. Rev. Lett. **50**, 1946 (1983).
[19] H. Rieger and A. P. Young, J. Phys. A **26**, 5279 (1993).
[20] H. Rieger, Phys. Rev. B **52**, 6659 (1995).
[21] M. Gofman, J. Adler, A. Aharony, A. B. Harris, and M. Schwartz, Phys. Rev. B **53**, 6362 (1996).